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# Evaluation of Particle Swarm Algorithm Performance In Optimal Portfolio Selection

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**ABSTRACT:** This research aims to evaluate the optimum portfolio selection using with particle swarm algorithm. For this purpose, the financial information of companies listed on the Iran stock exchange, during years 2007 to 2012 is collected and using heuristic particle swarm algorithm and based on Markowitz model, mean-variance model and client risk model, generating optimal portfolio from the stocks has been investigated. In total, the results of this study showed that use of this algorithm can provide solutions both close together and close to optimality, and causes confidence of the investors' investment for making decisions. Also, based on the response obtained by performing several experiments it can be claimed that in Markowitz and mean-variance models can provide most optilam portfolio. In other hands, particle swarm algorithm is best in client risk model. Most observations reflect the fact that in the problems which are with complexity and size increases, the particle swarm algorithm perform better.

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Key words: Particle Swarm algorithm, Markowitz Model, Mean -Variance Model, Client Risk Model

### INTRODUCTION

The literature can be significantly financial portfolio composition or set of the stock chosen by the investment to the investment, he said. The portfolio optimization of stock problems in in early 1952 was taken into account. Two important components in the investment decision are risk and return of the capital assets. Modern The portfolio theory, which was first introduced by Markouitz paradigm organizing the formation of the portfolio with the highest and Expected Returns on a given level of risk or set up an efficient set. According to Markouitz's theory, one can by minimizing risk for a given level of return on investment to a minimum variance portfolio. In this case, a new approaches, using of optimization initiatives.

Heuristics that aims to overcome the shortcomings of classical of optimization were introduced, the exhaustive search and random, the probability to achieve better results largely guarantee. Today, according the limited resources and the risk of investing in of financial assets, one major problem is that the utility of each investment to determine a set of stock portfolio the majority of it is too much. this is equivalent to selecting the optimal The portfolio of possible and the investment The portfolio of investors in the set a capital asset ment that at the lowest risk, highest efficiencies have for for this purpose, to should be on a model of optimal the portfolio choice in can help.

Already several patterns of are presented for solve the problem optimal portfolio according to conditions and limitations of each end are designed. Although theoretically as these models are solved using mathematical programming, but in practice there are problems with this area. Ment fund managers also in practice some of the limitations exercised upon their optimal portfolio that also cause this the problem more complex. Due to problems today ultra schematize Initiative of excessive method of solving optimization problems formation is taken into consideration. Indeed, choosing a financial portfolio in order to maximize efficiency, one of the major concerns of investors in financial markets. Actually, choosing a financial portfolio in order to maximize efficiency is one of the major concerns for investors in financial markets. The goal of this optimization is to determine the the allocation of financial capital in a way that yields the the maximum total assets and a risk is minimal.

The classical methods in selecting the optimal portfolio is generally not efficient enough, and now solve this problem, heuristic algorithms, including algorithms for of collective intelligence and a genetic algorithms, have been also considered. Collective Intelligence or groups of particles can optimize the portfolio to maximize returns and a minimize risk of investment to solve. This algorithm can be as algorithm as the core population and improve the long history of study in literatures of Optimization. Accordingly, given the importance of this issue, the present study aimed to evaluate the performance of particle swarm optimization portfolio (PSO) selection with respect to various constraints to investment in the stock portfolio.

In this context, this paper has five sections. The second section investigates the background of the research, including theoretical background and experimental research focus background. In the third

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section, the research methodology have been presented in which the model used this study and the methods of analysis are discussed. The fourth and a final section deals with the experimental results of this research study is devoted to the conclusions and presenting suggestions.

**Background Research:** Markowitz (1952), the fundamental model of modern portfolio theory was offered portfolio. The Markowitz mean-variance model (MV), showed the formation of a basket of assets, there is financing is the possibility that a certain level of efficiency and reduced risk. This model was originally proposed for the measure risk taking. The optimal portfolio selection problem in two, appropriate model selection and efficient and effective method to achieve the optimal solution are very important. Usually, the traditional mathematical methods and algorithms for solving these models not very good and accurate solutions for this kind of problem, mathematical programming algorithms effective and efficient programs exist (Gonez and Fernandez, 2007).

Groups of particles algorithm as a method without any derivative information the spaces and a complicated problems and derive an effective functioning and has high the convergence speed, strong, flexible leaves. Of Aspects computational systems algorithm and a not expensive cost, even with very modest memory and processors can be found in most home computers, is applicable. Algorithm and groups of particles or flock with of initial population is generated randomly, the beginning and complete the end bet arrives.

Terminating condition for algorithm for perform 100 successive iterations of algorithm. The algorithm requires every solution of the problem of Initiative of excessive simple form and can be used in programming is coding. Answer codec a significant impact the the speed and accuracy every algorithms is Initiative of excessive. Answer a title must be the association spanning between one to one and a answer the question of how to represent the solutions exist. In other words, every answer to the question exactly the same structure is displayed and the display is only one response to the corresponding question. Also, any response must be stored a small memory space. View every Answer is chosen so as operator and neighborhood requirements of the algorithms Initiative of excessive easily be done.

For coding the each selected solution portfolio, from an array of length the number of tasks to be used. This introduction is the fact that what the stock and what weight are selected. Ultra schematize particle swarm algorithm flowchart is presented in diagram 1. Meta-heuristic algorithm and particle swarm where each particle in is updated by the following formula:

 $X_i = Lower + (Upper - Lower) * random$ 

$$X_{i}(k+1) = X_{i}(k) + V_{i}(k+1)$$
 (2-1)

In the learn-heuristic algorithm and particle swarm, a population of particles randomly given initial position and velocity is choised. The best position the particle has so far (P-best) and the best position that the particles now have the whole set (G-best), is calculated at the the current stage. Third, the speed and location of every particle in the new stage of formula 1-2, and the final update, the stopping criteria are met, otherwise the algorithm and stops and will run Step 2 again. The main objective of this replacement is give local optimum and an optimal distribution of data among birds. So that, the birds in its path to be used. If Si, t and Ti, t as birds are considered mid locations, concepts chart below Vit, (Pit-Xit) and (Git-Xit) will map.



Figure 1. View of concepts particle swarm algorithm

At each step of the algorithm and particles for each of the members of the community, given the current status of its members, the position of its members and its position relative to the rest of society, distinct policies to create a new neighborhood for its members to adopt are. Choosing the right policy for the current position of each member of the parameter, select the appropriate policy for the member position relative to other members of the community, and to select appropriate policy parameters with respect to the the parameter  $FI_i(k)$  its member.  $II_i(k)$  Can be calculated. Then, according to these parameters distinctive policy is adopted. For each community the parameter  $FI_i(k)$  is obtained as follows:

$$FI_{i}(k) = 1 - \frac{f(X_{ik}.(k))}{f(X_{i}(k))}$$
$$FI_{i}(k) = 1 - \frac{f(X_{ik}.(k))}{f(X_{i}(k))}$$

(2-2)

Where the objective function objective function value of each solution, here is the latest completion time are important. If it is  $i = i_k^*$ , the second available as the value  $FI_i(k)$  is equal to zero. Possible factor in the case of  $f(X_{ik}.(k))$  is equal to zero, will be defined according to the problem would be impossible. Therefore, it would be  $0 < FI_i(k) < 1$ . Different politics according to the quantity of  $FI_i(k)$  is as follows:

$$MP_{i}^{current} = \begin{cases} Apply transportation & FI_{i}(k) = 0\\ Apply insertmutation & 0 < FI_{i}(k) < 1\\ Apply cross - over & F < FI_{i}(k) < 1\\ (2-3) \end{cases}$$

$$MP_{i}^{current} = \begin{cases} Apply \ transportation & FI_{i}(k) = 0\\ Apply \ insertmutation & 0 < FI_{i}(k) < 1\\ Apply \ cross - over & F < FI_{i}(k) < 1 \end{cases}$$

Exchange mutation operator used above, the randomly selected location and then actually doing the work of the position to another position in the matrix of the exchange is kept in the answer, the exchange of. *insert* The location of the mutation operator randomly selected and then it is transferred to another position to another position within the. Cross-Over Mutation operator is also the location of a randomly selected set of neighboring solutions that are exchanged and those with the best deals. The selection policy is based on the location of other members of the human motion to generate a predefined neighborhood of attempted to define batch size as category (X). For example, from 4X + 1 in each category, the number X + 2 is the next category is 2X + 1. In this case a bird in a batch is identified based on the mating combination, or should the best in each category occure (Raei, 2010).

Because of the way only able to generate random answer will examine the problem space and therefore cannot be completely sure that it was the general optimality. In other words, the number of repetitions is limited, only local optimality can be achieved. Therefore, it is necessary to determine the conditions, the total amount assessed optimality. The boundaries specified performance is presented in diagram 2.

Accordingly, when the risk is minimal, portfolio return for portfolio selection and great value during these two together near the value of each selection is also more balance of the value of a much pass specifically, the increased risk to be addressed. In other words, in this case, compliance risk, high return on the investor for a greatly increased - increased.

To evaluate the performance of the proposed algorithm and,  $\lambda$  to determine the amount and on the other hand, continuous changes and experimental performance of each algorithm and, the amount of this difference is unknown. To this purpose, level  $\lambda$  than any of the previous motion be 0.05 increase in the number of 20 different tests are carried out for  $\lambda$ .



Figure 2. Standard efficient frontier

**Empirical research literature:** In recent years, the portfolio optimization is considered in empirical studies, however, using the technique of aggregated particles in comparison to other methods of combinatorial optimization under consideration is. In this case, Pngyng and Vanek (2006), in their study, using the particle swarm nonlinear resource allocation efficiency of this method have been compared with genetic algorithm. Overall, these results indicate that the particle swarm algorithm and is more efficient than genetic algorithm.

The results Jyam and followers (2008), combining genetic algorithm and optimization techniques based on particle mass Memetics algorithm and in which the technique only applies to on the answers obtained by genetic algorithm, and show that using this algorithm, groups of particles portfolio much more efficiently than when the algorithms are applied separately. Tufchang (2009), in other studies, the efficiency of groups of particles optimization problem constrained portfolio during the period from 1992 to 1997, has been tested. The results of this study suggest that this technique is very successful in portfolio optimization.

Studies conducted in-country stock portfolio optimization heuristic optimization technique has received less attention. In this regard, Raie (2010), in their study, the stock portfolio optimization problem using Particle Swarm moves in 20 companies in Tehran

Stock Exchange during the period 2006-2008 is discussed. Overall, these results indicate that the method of Particle Swarm Optimization portfolio equity restrictions have been successful. Nikzad (2011), in other research, genetic algorithms, Simulated Annealing Memetics and the portfolio optimization of the shares on the stock exchange, has been compared. Overall, the results indicate that the proposed algorithm is capable of Memetics portfolio optimization problems with risk criteria, taking into account the limitations of integer for the number of stocks in the portfolio to solve. The results show that the algorithm Memetics in all cases studied, the best results obtained by Genetic Algorithm and Simulated Annealing are presented.

#### MATERIAL AND METHODS

The present study, using particle swarm optimization portfolio and compare the results, in terms of risk and return. For this reason, financial data, 30 companies in Tehran Stock Exchange regarding financial intermediaries, non-metallic mineral Other products, automobile and parts manufacturing during the period 2007 to 2012 with the aim of collecting and identifying the optimal portfolio consists efficient frontier investment to identify and establish the optimum portfolio using the heuristic algorithms are studied.

The determination of the portfolio, in the form of study models, assumptions, and there are several variables that must be considered. Markouitz model, all choices are separate and independent from each other they are observed. Also, every choice has a defined benefit and could be due to differences with the other options outlined. Each of the designs can range from zero to 100 percent of the capital stock of its design capacity of one hundred percent of their total weight must be selected to form a complete basket of. The first model of Markowitz mean-variance risk measure for measure ¬ bid. Classical model mean-variance is as follows:

$$Min \sum_{i=1}^{k} \sum_{j=1}^{k, j \neq i} x_i x_j \sigma_{i,j}$$
  
$$S t : \sum_{i=1}^{k} x_i \mu_i = R$$
  
$$\sum_{i=1}^{k} x_i = 1$$
  
$$0 \le x_i \le 1$$

(3-1)

 $\mathcal{X}_i$  In which the amount of capital held in the plan  $\mu_i$  i, i, and the expected value per unit of the scheme  $\sigma_{i,i}$  is the variance between the two designs i and j. In this model, in line with the objective function, the objective is to minimize the risk of. Risk in this case there is a large divergence between selected elements in the basket. The main difference is that the display is the standard deviation: the more increases the risk of. Fernandez and Gomez (2007), the Markowitz model with the addition of upper and lower limits for variables modified models mean-variance components presented tying (CCMW). General form of the model of Fernandez and Gomez, to as follows:

$$Min \sum_{i=1}^{k} \sum_{j=1}^{k, j \neq i} x_i x_j \sigma_{i,j}$$
  
$$S t : \sum_{i=1}^{k} x_i \mu_i = R$$
  
$$\sum_{i=1}^{k} x_i = 1, \varepsilon_i \le x_i \le \xi_i$$
  
(3-

2)

Where the first capital of making the scheme i  $\mathcal{X}_i$ , expected value per unit of the scheme i  $\mu_i$ ,  $\sigma_{i,j}$  degree variance between project i and j,  $\mathcal{E}_i$  the lower limit for the selected stock of i,  $\xi_i$  and the upper limit for the selected stock of the i's. In this model, the first model is established Mfrzvat condition but in fact - will have a more realistic. In this situation no longer is a plan ever wanted to buy or plan to buy some did not. In these situations a certain extent should be considered as the main range. The main disadvantage of this method is limited, failing to optimize portfolio selection problem under the constraint of integer constraints.

Because in the the real world and the real financial decisions often require investors to determine the exact number of assets in their portfolio.Customer risk models entering the integer restrictions, the model closer to the the real world and thus also solving practical and useful decisions in the hands of the investors.Login restrictions programming integer programming and nonlinear discrete space continuous search space will become. This situation causes Kuadratyk integer linear programming is an open compound. Integer restrictions are added to the model as follows:

$$Min \lambda \left[\sum_{i=1}^{k} \sum_{j=1}^{k, j \neq i} z_{i} z_{j} x_{i} x_{j} \sigma_{i, j}\right] - (1 - \lambda) \sum_{i} z_{i} x_{i} \mu_{i}$$
  

$$S t : \sum_{i=1}^{k} x_{i} = 1$$
  

$$\sum_{i} z_{i} = K$$
  

$$\varepsilon_{i} z_{i} \leq x_{i} \leq \xi_{i} z_{i}, z_{i} \in [0, 1]$$
(3-3)

In model  $\lambda$ , the risk of taking or risk aversion shows. In this model, the Type  $\lambda$  in the objective function, and returns both Myarrysk Drtab goal arrived

#### RESULTS

The results of the estimation of the efficient frontier of particle swarm algorithm in A Markovitz model, constrain mean-variance and customer risk models, which are presented in Figures 3 to 6, show that the algorithms particle swarm with good accuracy can solve the problem of portfolio optimization. In Markovitz model, toward to constrained model of mean-variance, that the addition constrain of weight is equal to one, for investment limit is imposed, Particle swarm algorithm have best performance and distict to efficient frontier is less.

In contrast, when that the limited number of customer risk model asset portfolio will be added to the model, particles swarm distance from the efficient frontier obtained with the standard groups of particles algorithm is more and its efficiency should low.

In other words, most observations reflect the fact that in the problems are smaller and more lightweight, groups of particles algorithm is best and have better performance, but in the problems are larger and more with increasing sample size, this ways is not efficient. Therefore, particles swarm algorithm isnt efficient in and while Hdaqlsazy risk, maximizing efficiency is considered. In fact, it is a weighting the parameter whose value varies in the range of [0-1] values reported by the investor to the risk or return are applied. In other words, the higher  $\lambda$  , the more important efficiency and simultaneously reduce the quantity  $1 - \lambda$  weight can lower the risk minimization objective. These models need to consider the personality of the investor's capital have been presented and is Consider an investor can expect a different attitudelf the person is a flight risk, , and ( $\lambda$ ) an amount equal importance to reduce the risk of a will. On the other hand, the venture capital more, this quantity tends to zero so that the second term of the objective function given more power and look profit maximization is the max.

the third model. Power of this algorithm be compared with the pre prediction algorithm and is presented In the Table 1.

In this table, the performance of this algorithm and in the terms of objective criteria and is considered the difference in the final period. In other words, the main criterion for evaluating algorithm based on the belief that algorithm and with respect to the previous period, to what extent it has been able to determine a basket in the final period, to make maximum profits. For this purpose, the data set selected for the basket final period, the percentage of each share in the basket so that maximum benefit is derived if it is.

Moreover, the estimated objective function in the each group of particles algorithm based on Markowitz model, constrained mean-variance and client risk model is presented in the diagram 6. These results also show that this algorithm, in models of Markovitz, meanvariance is more efficient. The algorithm and handles the movement of birds in the one and two models results closer to optimality offers. The third model Estimated error rate is presented In the Table 2.



Diagram 3. standard efficient frontier and the performance of groups of particles algorithm in Markowitz model



Diagram 4. Standard efficient frontier and the performance of groups of particles in mean - variance bound model



Diagram 5. Standard efficient frontier and the performance of groups of particles algorithm in customer risk model

Groups of particles algoritm							
Customer risk models		Model mean - variance constrained		Markowitz model			
Difference with the the Courses	The objective function	Difference with the the Courses	The objective function	Difference with the the Courses	The objective function		
0.046	394	0.028	324	0/044	214		
0.05	227	0.016	215	0.086	213		
0.076	225	0.042	211	0.077	200		
0.061	283	0.018	283	0.017	279		
0.052	215	0.013	213	0.087	213		
0.054	360	0.042	325	0.096	290		
0.08	213	0.079	207	0.032	204		
0.042	267	0.013	247	0.018	226		
0.044	202	0.019	202	0.015	202		
0.05	207	0.065	203	0.037	203		
0.014	281	0.066	237	0.095	222		
0.075	311	0.08	263	0.053	232		
0.075	389	0.008	310	0.002	308		
0.051	226	0.02	214	0.055	202		
0.064	324	0.037	287	0.095	249		





**Diagram 6**. Estimate of the objective function of particles Swarm algorithm

Type of model	Groups of particles Algorithm		
Index	The average	The variance	
Markovitz	0.056	0.0012	
The average - variance constrained	0.043	0.00071	
The customer risk	0.057	0.00045	

**Table 2.** Comparison of mean and variance of the error of particles swarm algorithm

#### DISCUSSION

In this study, performance of particle swarm algorithms based on three models Markovitz, constrain mean-variance and customer risk for portfolio optimization has been evaluated. These results suggest that in constrained mean-variance and Markowitz models, the distance of the efficient frontier obtained with the standard frontier in particles algorithms, was lower, while the differences with the standard efficient frontier, is more in risk model and the client has not a efficient performance. Dominant of the research on groups of particles algorithms in comparison with other studies also confirmed that the concept of groups of particles algorithms is that it can be well on rapid and accurate analysis of the issues. Cause of the claim, there must be a group attitudes to similar cases previously considered.

In response to a possible hypothesis optimization ability of groups of particles algorithm can be said, the optimal portfolio selection problem increases with greater complexity constraints, it is not only appropriate an method but not to be the suitable the complexity of dealing with their own show. In other words, most observations reflect the fact that while the problems are smaller and lighter than with the increasing complexity and size, groups of particles algorithms have better performance. Groups of particles algorithms in the Markovitz model, the faster will draw efficient frontier, although this algorithm as compared to Mean-Variance more tender time variance bound for algorithms is necessary. The following suggestions can be used to evaluate data in order to improve the tools of financial analysis and can be expressed as:

1. Development of other innovative methods such as Ant colony algorithm, electro-magnetism, and harmonies of music, Memetics algorithms and Simulated Annealing technique to study the optimal portfolio.

2. Adding catalog others investment restrictions investment of the mathematical model and its solution using innovative methods

3. Use of algorithms studied in this research, optimization and comparison of companies in the Stock Exchange Market and applies of these results to guide investors.

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