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Some Conceptual Difficulties of Students on Derivation

Dejene Girma Denbel
Department of Mathematics, Dilla University, Ethiopia
*Corresponding author's Email: dejene28@gmail.com

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ABSTRACT: The topic derivation is one of the fundamental concepts in the learning of calculus in university. It is a prerequisite for other concepts in that level and its traces are visible in the majority of mathematical courses at university level. Students have difficulties in the learning of this concept which mostly come back to lack of conceptual understanding and focusing only procedural aspects. The purpose of this study was to investigate conceptual difficulties that these students faced, to understand derivation conceptually. The design of this study is qualitative analysis of open-ended questions, and its subjects consisted of 60 university students. The findings showed students have serious difficulties in understanding derivation conceptually. The students' responds indicated that main reasons of difficulties in conceptual understanding of derivation come back to focusing on symbolic aspect more than embodied aspect (like graph), lack of making logical connection between these aspects, and weakness of dealing with generalized question. Findings of this study provided information to calculus instructors and students to overcome learning difficulties of derivation.

Key words: Derivation, Difficulties, Conceptual Understanding

INTRODUCTION

The calculus represents the first time in which the student is confronted with the limit concept, involving calculations that are no longer performed by simple arithmetic and algebra, and infinite processes that can only be carried out by indirect arguments. Teachers often attempt to circumvent the problems by using an "informal" approach playing down the technicalities. However, whatever method is used, general dissatisfaction with the calculus course has emerged in various countries round the world in the last decade (Tall, 1992). There are some difficulties identified by researchers' worldwide regarding calculus. Particularly limit derivation and integral.

Restricted mental images of functions

It is not always seen as provoking a difficulty in elementary calculus particularly when the subject is seen as focusing on the differentiation and integration of standard functions given as formulae. Nevertheless it causes difficulties as soon as the student is faced by examples slightly beyond their experience, such as calculating $\int_{-3}^{3}|x+2|\,dx$ Mundy (1984) or finding a, b such that $f(x)=\begin{cases} ax & , x\le 1 \\ bx^2+x+1, & x>1\end{cases}$ is differentiable at 1 Selden et al. (1989), then the students fare extremely badly. Unless students meet the concept of function in a broader context, such difficulties should be expected.

Difficulties in translating real-world problems into calculus formulation

This is part of the folk-lore of the subject (though there seems to be little cognitive research). Many examinations for calculus examinations focus on the symbolic manipulation rather than problem-solving (see, for example, the selection of examination papers quoted in Calculus for a New Century (Steen,1988).

The Leibniz notation $\frac{dy}{dx}$

It proves to be almost indispensable in the calculus. Yet it causes serious conceptual problems. Is it a fraction, or a single indivisible symbol? What is the relationship between the dx in $\frac{dy}{dx}$ and the dx in $\int f(x)dx$? Can the du be cancelled in the equation $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$? Giving a modern meaning to these terms that allows a consistent meaningful interpretation for all contexts in the calculus is possible but not universally recognized. On the other hand, failing to give a satisfactory coherent meaning leads to cognitive conflict which is usually resolved by keeping the various meanings of the differential in separate compartments ($\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}$ in differentiation and dx means "with respect to x" in integration). This can only exacerbate conceptual chasm between the notation and any possible coherent meaning.

Difficulties in selecting and using appropriate representations

Robert & Boschet (1984) reported that the students who were the most successful were invariably those who could flexibly use a variety of approaches: symbolic, numeric, visual. Dreyfus & Eisenberg (1986, 1991) report students' reluctance to visual concepts in calculus. They give examples where visual representations would solve certain problems almost trivially, yet students refrain from using them because the preference developed over the years is for a numerical, symbolic mode of approach. Yet research shows that visual images can provide vital insights. However, it may sometimes prove difficult for students to link the global gestalt to a sequential deductive form of thinking. The concept of derivative is considered difficult for most undergraduate students (Tall, 1993, 2011; Willcox and Bounova, 2004; Metaxas, 2007; pepper et al., 2012). Students' difficulties in learning of derivation are caused by their lack of conceptual understanding (Tall, 2011). According to many researchers students' conceptual understanding is not sufficient in the learning of mathematics (Tall, 1992, 2012; Stacey, 2006; Metaxas, 2007).

There are some methods being introduced to support students to overcome their difficulties in the learning of derivative. Researchers endeavors to support students in learning of derivative concept by promoting mathematical thinking with or without computer assistance. Dubinsky (1991) conducted research that promoting mathematical thinking to help student understanding in calculus in general and derivation in particular.

This research indicated that derivation is seemed difficult for students who have found difficulties the calculus concept. The main objective of this study was to know what kind of difficulties these students have of the concept of derivative.

MATERIAL AND METHODS

The subjects of this study were 60 selected based on their availability without using any sampling method from first year students attending at the college of natural and computational Science of Dilla University in Ethiopia in 2014. These students took Calculus I/applied mathematics I in their first semester. Calculus I/applied mathematics I course include the concept of derivation.

For investigation of conceptual understanding on derivation, 4 question of conceptual understanding of derivation in the study. The questions of conceptual understanding have been designed based on definition of conceptual understanding described by Haber and Abboud (2006) and ideas and theories from researchers such as Orton (1983a,1983b), Tall (2004, 2008) and others about understanding of derivatives conceptually were used.

Questions of Conceptual Understanding in derivation

1. Define the relative maximum and minimum and absolute maximum and minimum. Show also your definitions using graph.
2. Show how the turning point can be found by using graph and figure, and assert that how it can be symbolically expressed.
3. In the question of motion, what is the difference between average velocity (ΔV) and velocity moment? Can velocity moment be shown by using average of velocity (ΔV)? Show them using Shape and figure.
4. Based on graph and shape of function, how can you say that a function is increasing or decreasing?

RESULTS AND DISCUSSION

Question-1: in this question students were asked to define the relative and absolute maximum and minimum. Moreover, they were asked to show their definitions using graph.

Out of 60, 37 students (62%) couldn't answer it at all gave without response. 9 students (15%) tried to give the meaning of maximum and minimum. they defined maximum means the highest point and minimum means the lowest point. these students didn't realized and gave attention the meaning of their saying in calculus. That means they were not able to make connection between the meaning of these expression and their interpretations in the calculus. Moreover, these students didn't used figures to explain maximum and minimum.

Only 5 students out of 60 (8%) have given right definition of maximum and minimum in calculus .but these students didn't explained their answer graphically.

Therefore, a total of 14 students tried to give definition of maximum and minimum (9 not in connection with calculus and 5 correctly in connection with calculus).

Their understanding can be summarized as:

"Absolute maximum as highest and absolute minimum as lowest point and relative minimum and maximum should be selected on ordering bigger to smaller"

Among students 5 students out of 60 responded this question by using only figure and graph .they couldn't connect graph of maximum and minimum to their algebraically.

That means these students were able to interpret these terms based on their shapes and figures.

Only 9 students (15 %) gave a complete answer for question -1. They could also use both algebraic and graphical answer to the question.

- They have shown maximum and minimum by use of graph,
- They able to interpret the properties of maximum and minimum of graphs symbolically,

Graph of One of students' (model) is shown below.

Question-2: in this question students were asked to show how the turning point can be found by using graph and figure, and assert that how it can be symbolically expressed.

Question-3: In this question students were expected explain the difference between average velocity (ΔV) and moment velocity and to explain their understanding using Shape and figure. Mainly this question assesses their understanding on the role of derivative in the moment velocity /instantaneous velocity.

Question-4: in this question students were requested how to identify a function is said to be increasing or decreasing based on graph and shape of function. In this question all students were responded.

Table 1. Summary of students' response and difficulties on question-2

Number of students	Response	Major difficulties
42	Didn't give any response	No understanding at all
10	Tried to give meaning of turning point graphically but it is wrong	Problem of visual /graphical representation of concept as mentioned by Robert & Boschet (1984)
3	They wrote the correct answer but it is in sentence for not graphically $f'(c) = 0$ and f' changes sign around c thenc is turning point of f	Luck graphical meaning of turning point
5	Gave definition, graph and algebraic aspects of turning point(which is a complete answer)	Have no any difficulty on turning point conceptually

Table 2. Summary of students' response and difficulties on question-3

Number of students	Response	Major difficulties
32	Didn't give any answer at all	No understanding at all
12	Explained about average velocity but not on moment velocity. Wrote formula for average velocity $V_{av} = \frac{v_2 - v_1}{t_2 - t_1}$ not say anything on velocity moment	Unable to associate derivative with moment velocity. Restricted mental image of derivation
11	Responded that no difference between average velocity and moment velocity	Not have full understanding
5	Gave clear definition for both. Demonstrated the difference between the two. Mentioned the role of derivative to find moment velocity from average velocity.	No difficulties

No students used to explain the two velocities graphically.

Table 3. Summary of students' response and difficulties on question-4

Number of students	Response	Major difficulties
27	Defined increasing and deceasing function.	They know increasing is opposite of decreasing. (Interpreting as terms was lexical). No any connective understanding of increasing and decreasing with derivative
13	Understood the ways of diagnosing increasing and decreasing function using derivative. $f'(x) > 0$ increasing and $f'(x) < 0$ decreasing	Unable to explain their understanding graphically. Similar to result on (Robert & Boschet ,1984)
11	Gave only graphical explanation of increasing and decreasing function.	Unable to mention algebraically meaning mainly use of derivative.
9	Gave both algebraic and graphical meaning and properly explained the role of derivative. It shows only these students come up to this question correctly.	No difficulty observed

DISCUSSION

One can easily observe that only limited number of students in each of 4 conceptual understanding of derivative question responded correctly. This shows these students have difficulties in the learning of derivation. In other word there was a lack of conceptual understanding among undergraduate students who take part in this study.

Based on the data the following difficulties are observed:

- ❖ Not using both graphical and algebraic aspects at same time,
- ❖ Weakness of making relationship between these aspects and focusing on algebraic aspects more than graphical aspect.
- ❖ Students have difficulties to deal with general postures of derivative's concepts based on the results of qualitative analysis.

In the end, the difficulties of conceptual understanding of derivative can be summarized in to two main categories:

❖ Students do not pay attention to the importance of both embedded and symbolic aspects of derivatives in the learning of this concept. Focus more on symbolic than embedded or graphical aspect.

❖ Lack of making connection and relationship between embedded and symbolic aspect.

CONCLUSION

It would appear that there is a strong necessity to find suitable strategy to cover most of these difficulties for improving students' conceptual understanding and problem solving abilities within the learning and teaching of derivative. This result also helps for other researcher to select and design effective teaching techniques to overcome the mentioned difficulties.

Implication

The results of the study have shown that students the majority of students in the study lack conceptual understanding about derivation. It seems these students only focus the procedural aspects of derivation rather than conceptual understanding of derivation. It shows how our teaching of derivation rest on procedural teaching only.

Calculus instructors who are teaching this course shall understand students' gap and modify they teaching by shifting from traditional to conceptual teaching approach.

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